BTM/UMC/UMA-501   Roll No. ...153...
BNT/UNT-502

**B. TECH (MAE, NS & T), B. TECH (MAE) +
M. TECH (A) – DD, B. TECH (MAE) + MBA,
B. TECH + M. TECH (NS & T) – DD &
B. TECH (MAE) – EVENING**

FIFTH SEMESTER END TERM EXAMINATION :
NOVEMBER, 2013

DISCRETE MATHEMATICS

Time : 3 Hrs.   Maximum Marks : 70

Note: *Attempt questions from all sections as directed.*

SECTION – A   (30 Marks)
Attempt any 5 questions.
Each question carries 6 marks.

1. (a) Show that the truth value of $(p \wedge (p \to q)) \to q$ is independent of their components.

   (b) Prove that the premises $p \to (q \to r)$, $s \to (q \wedge \sim r)$ and $p \wedge s$ are inconsistent.

2. Solve the recurrence relation
   $$a_n - 5a_{n-1} + 6a_{n-2} = 2.3^n (n + 1)$$

3. Simplify $E(x, y, z, t) = \sum (1,2,4,5,6,11,12,13,14,15)$ using K-maps.

                                                  P.T.O.

4. Let Z be the set of integers and R be the relation defined by
$$R = \{(a, b) : a = b \pmod 7\},$$
Show that R is an Equivalence relation. Also determine equivalence classes generated by the elements or Z.

5. (a) Show that $x^2 + 5x + 11$ is $O(x^3)$.

   (b) Among first 200 positive integers, how many integers are neither divisible by 2, nor by 3, nor by 5.

6. (a) Show that the relation "parallel" is not partial order relation on the set of lines.

   (b) If A and B are two subsets of universal set, then prove that
   $$A - B = A \text{ iff } A \cap B = \phi.$$

SECTION – B   (20 Marks)

Attempt any two questions.
Each question carries 10 marks.

7. (a) Let f, g and h be functions from N to N, where N is the set of natural numbers so that
   $$f(x) = x + 1$$
   $$g(x) = 2x$$
   $$h(x) = \begin{cases} 0 & \text{if } x \text{ is even} \\ 1 & \text{if } x \text{ is odd} \end{cases}$$
   Determine (fog)oh.

   (3)

+ (b) Show with an example that the union of two sublattices may not be a sublattice. (3)

(c) Out of 7 consonants and 4 vowels, how many words can be made each containing 3 constants and 2 vowels. (4)

8. (a) Consider the lattice $D_{50} = \{1,2,5,10,25,50\}$ ordered by divisibility.

   (i) Draw the Hasse Diagram of D_{50}.

   (ii) Which elements are join irreducible elements and atoms ?

   (iii) Find the complement of 5 and 10. Is it complemented lattice.

   (iv) Determent the greatest and least element of D_{50}. (6)

(b) Prove that if x is irrational then $1/x$ is irrational. (4)

9. (a) The nth Fibonacci number denoted by F_n is related by the recurrence relation

$$F_{n+1} = F_n + F_{n-1}, \; n \geq 2 \text{ and } F_1 = F_2 = 1$$

Use Strong Mathematical Inductive prove that

$$F_n \leq \left[\frac{1 + \sqrt{5}}{2} \right]^{n-1}, \quad n = 1, 2, 3 \ldots \ldots \quad (6)$$

P.T.O.

(b) Check the validity of argument of following argument : "If Anil is married, he is sad. If he is sad then he does not watch TV. He does watch TV. Therefore, Anil is unmarried."   (4)

SECTION – C   (20 Marks)

(Compulsory)

10. (a) Using Generating function, solve the Recurrence Relation $a_n - 4a_{n-2} = n$, $a_0 = 0$, $a_1 = 1$; $n \geq 2$.

(4)

(b) With or without using truth table, find PCNF and PDNF of $(\sim p \to r) \wedge (q \leftrightarrow p)$.   (6)

(c) Let $B = \{1, 5, 7, 35\}$ be the set of positive integers and operations '+' and '.' are defined as :

$a + b = 1\,cm\,(a, b)$ and $a \cdot b = gcd\,(a, b)$ for all $a, b \in B$

A unary operation on B is defined as $a' = 35/a \;\forall\; a \in B$

Show that $(B, +, ., ')$ is a Boolean algebra.   (5)

(d) Use K-map to find minimal sum of Boolean Expression

$f(x,y,z) = \sum(1, 2, 4, 5, 6, 11, 12, 13, 14, 15)$ (5)

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